

128 PRIME NUMBERS and PRIME FACTORS V2

Prime numbers

A prime number can be divided without a remainder only by itself and 1. For example, 17 can be divided only by itself and 1 without a remainder, and is therefore a prime number. A composite number is a number that can be divided by another number without giving rise to a remainder. The following points are useful in determining whether a number is a prime.

- The only even prime number is 2. All other even numbers can be divided by 2
- If the sum of a number's digits can be divided by 3, that number is divisible by 3 without a remainder
- No prime number greater than 5 ends in a 5. Any number greater than 5 that ends in a 5 can be divided by 5
- Zero and 1 are not considered prime numbers
- Except for 0 and 1 a number is either a prime number or a composite number. A composite number is defined as any number greater than 1 that is not a prime. For example, 4, 6, 8, 9, and 10 are composite numbers

The table on the next page shows the prime numbers up to 997. The earliest primes are: 2, 3, 5, 7, 11, 13, 17, and 19. All composite numbers are the result of multiplying two or more primes.

Prime factors

The prime factors of a composite number are the primes that when multiplied together give that number. The primes of the composite numbers from 1 to 9 are;

$$4 = 2 \times 2$$

$$6 = 2 \times 3$$

$$8 = 2 \times 2 \times 2$$

$$9 = 3 \times 3$$

To find the prime factors of any number we first divide by 2 for as many times as possible (for an odd number there will be no possibility of dividing by 2); we then divide successively by 3, 5, 7, 11 and all subsequent primes for as many times as we can until the result of our dividing is a prime. For example, the prime factors of 48 are:

$$2 \times 2 \times 2 \times 2 \times 3$$

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67
71	73	79	83	89	97	101	103	107	109	113	127	131	137	139	149	151	157	163
167	173	179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269
271	277	281	283	293	307	311	313	317	331	337	347	349	353	359	367	373	379	383
389	397	401	409	419	421	431	433	439	443	449	457	461	463	467	479	487	491	499
503	509	521	523	541	547	557	563	569	571	577	587	593	599	601	607	613	617	619
631	641	643	647	653	659	661	673	677	683	691	701	709	719	727	733	739	743	751
757	761	769	773	787	797	809	811	821	823	827	829	839	853	857	859	863	877	881
883	887	907	911	919	929	937	941	947	953	967	971	977	983	991	997			

Exercises: find the prime factors for:

1. 68, 90, 174, 106, 110, 142, 144, 155, 133, 192.
2. 52, 96, 91, 78, 66, 76, 128, 105, 186, 112
3. 138, 92, 165, 184, 171, 189, 80, 154, 126, 125
4. 130, 65, 112, 92, 182, 171, 154, 60, 188, 147
5. 231, 252, 126, 175, 196, 187, 104, 186, 217, 222

Simplifying fractions

If we are required to simplify fractions, we will find that the process of prime factorization will help us. Suppose the fraction to be simplified is 120/165, Prime factorization of the numerator and denominator gives:

$$\begin{array}{l} 120 = \underline{2 \times 2 \times 2 \times 3 \times 5} \\ 165 = \quad \quad 11 \times 3 \times 5 \end{array}$$

Because $3/3 = 1$ and $5/5 = 1$, we can simplify the fraction to

$2 \times 2 \times 2/11 = 8/11$ This is much handier than the original fraction,

Notice that if either the numerator or the denominator is a prime you cannot simplify the fraction further. If the result is an improper fraction, convert it into a mixed number.

1. $\frac{42}{58}$ $\frac{36}{74}$ $\frac{93}{108}$ $\frac{84}{140}$ $\frac{180}{207}$ $\frac{36}{60}$ $\frac{126}{210}$ $\frac{390}{220}$
2. $\frac{152}{323}$ $\frac{187}{357}$ $\frac{130}{195}$ $\frac{115}{207}$ $\frac{224}{252}$ $\frac{171}{54}$ $\frac{126}{84}$ $\frac{104}{78}$
3. $\frac{171}{266}$ $\frac{189}{175}$ $\frac{276}{92}$ $\frac{286}{242}$ $\frac{222}{148}$ $\frac{252}{182}$ $\frac{272}{187}$ $\frac{184}{138}$